

Real Analysis, H.L. Royden and P.M. Fitzpatrick
Errata/Comments on Fourth Edition, First Printing¹

Last Edited on August 28, 2010

Preamble, Preface, and Preliminaries

(Preamble) **On the dedication page, include ‘To John Slavins, H.L. Royden’
This was inadvertently omitted.**

(Table of Contents) in the title of 15.2 change Helley to Helly

(Table of Contents) in the title of 19.2 change $p \leq \infty$ to $p < \infty$

(Table of Contents) in the title of 19.3 change $p < 1$ to $p < \infty$

(Preface; page X; line -12) Replace ‘1998’ by ‘1990’

(Preface; page X1) Replace ‘Helley’ by ‘Helly’, twice

Chapter 1

(Chapter 1; page 12; line -4) Replace ‘natural’ by ‘rational’

Chapter 2

(Chapter 2; page)

Chapter 3

(Chapter 3; page 63; line 2 and 6) Change ψ_n to ϕ_n

(Chapter 3; page 65) In the last two line of the proof of Lemma 10, change ϵ to δ , twice. Also, replace E_n by E_N .

Chapter 4

(Chapter 4; page 89) In formula (22) replace $f'(x)$ by $f'(x) < \infty$

(Chapter 4; page 95) On the line after (30), remove “23 and ”

Chapter 5

(Chapter 5; page 98) In the definition of tight, remove an a.

(Chapter 5; page 99) Interchange Problems 1 and 2.

(Chapter 5; page 100; line -2) Fix braces around Riesz

(Chapter 5; page 101; line 5) Replace f_{n_k} by $f_{n_k}(x)$

¹I thank Richard Hevener for almost all the typos in Part I—pmf

- (Chapter 5; page 101) In displayed equation (6), replace 0 by $f \equiv 0$
- (Chapter 5; page 103; line 4) Replace ‘real-valued’ by ‘real’
- (Chapter 5; page 104; line -16) Insert the word “are” before the word “upper”

Chapter 6

- (Chapter 6; page 110; line 2) Replace ∞ by n
- (Chapter 6; page 113; line 7) Relace “mathematics” by “mathematical”
- (Chapter 6; page 113) In equation (15) interchange the limits of integration
- (Chapter 6; page 113) In the footnote, change 8 to 9
- (Chapter 6; page 119) Change the conclusion of Problem 36 to: “on closed intervals that contain x_0 and have arbitrarily small length.”
- (Chapter 6; page 121; line 4) Replace TV by V.
- (Chapter 6; page 122) Remove the second comma in equation (26)
- (Chapter 6; page 123; line 2) replace 7 by 8
- (Chapter 6; page 123) In the displayed equation of Problem 38, replace the first ‘<’ with ‘< ϵ if’
- (Chapter 6; page 124) Interchange the limits of integration in (28)
- (Chapter 6; page 124; -4) Replace 95 by 94
- (Chapter 6; page 129) In part (vi) replace 0 by $g(0)$
- (Chapter 6; page 129) In Problem 57, replace “the last part” by “part (iv)”
- (Chapter 6; page 132; line -10) Interchange u and v in the quotient
- (Chapter 6; page 134) In Problem 71, replace 45 by 43, twice.

Chapter 7

- (Chapter 7; page 138; line 1) Replace “The positivity homogeneity properties follow” by “Nonnegativity and positive homogeneity follow”
- (Chapter 7; page 138; line -3) Insert f before \in
- (Chapter 7; page 139) In Problem 2, insert “be ” after “to ”
- (Chapter 7; page 139; line -4) Replace L^1 with L^p
- (Chapter 7; page 140) On the line after displayed equation (3), replace $L^q(X, \mu)$ by $L^q(E)$.
- (Chapter 7; page 140; line -8) Replace f by g , twice

- (Chapter 7; page 143) In the first example, replace $\alpha <$ with $\alpha \leq$
- (Chapter 7; page 143) In the second example, replace $E = (1, \infty)$ by $E = (0, \infty)$ and replace $\ln x$ by $|\ln x|$ and remove “for $x > 1$ ”
- (Chapter 7; page 143) In Problem 9, replace $a = b = 1$ by $a^p = b^q$
- (Chapter 7; page 143) In Problem 12, replace $1 \leq p < \infty$ by $1 \leq p \leq \infty$
- (Chapter 7; page 144) In Problem 20, replace $p < \infty$ with $p \leq \infty$
- (Chapter 7; page 144) In Problem 22, replace $1 < p < \infty$ by $1 \leq p < \infty$
- (Chapter 7; page 148) Richard Hevener observed that the General Dominated Converges Theorem provides a short, direct proof of Theorem 7
- (Chapter 7; page 148; line -8) Replace the second and fourth f_n by f
- (Chapter 7; page 149) Change f to f_n in the last line of the statement of Theorem 8
- (Chapter 7; page 150; line 15) Replace f_k by f_n , twice
- (Chapter 7; page 150; line -13) Replace f_k by f_n , twice
- (Chapter 7; page 150; line -2) Remove the comma before “provided”
- (Chapter 7; page 151; line -2) Replace “, and ” by “. We ”
- (Chapter 7; page 153) In Problem 39, replace $1 \leq p < \infty$ by $1 \leq p \leq \infty$
- (Chapter 7; page 154; line -10) Replace $\|\Phi(f) - \Phi(g)\|_p$ by $\|\Phi(f) - \Phi(g)\|_p^p$

Chapter 8

- (Chapter 8; page 155; line -7) Replace “,Helley ” by “Helly ”
- (Chapter 8; page 157) In the proof of Proposition 2, replace $q - 1$ by $1 - q$. The proof does not cover the case the case $p = 1$. For $p = 1$, argue by contradiction. If $|g|_\infty > |T_*|$, there is a set E_0 of finite positive measure on which $|g| > |T_*|$ and one gets a contradiction by choosing f to be $1/m(E_0) \cdot \text{sgn}(g) \cdot \chi_{E_0}$.
- (Chapter 8; page 158; line -3) Remove the second comma
- (Chapter 8; page 161; line -8) Replace Lebesgue-Stieltjes by Riemann-Stieltjes
- (Chapter 8; page 162) In Problem 10, replace $\text{TV}(f)$ by $\text{TV}(g)$.
- (Chapter 8; page 163; line 4) Replace Radamacher by Rademacher
- (Chapter 8; page 167) Change the font on the last line of Theorem 11
- (Chapter 8; page 167) In the Riemann-Lebesgue Lemma, replace “corollary” by “theorem”. Also, extend the lemma to $p = 1$ by using Theorem 10, the density of the simple functins in L^∞ , and Theorem 12 of Chapter 2.

(Chapter 8; page 167; line -3) Replace the second 1 by 0

(Chapter 8; page 168; line 15) Replace 11 by 10.

(Chapter 8; page 170) In Problem 17 (iii) replace the second f_n by f

(Chapter 8; page 170) In Problem 19, replace $1 \leq p < \infty$ by $1 < p < \infty$

(Chapter 8; page 171) Replace Helley by Helly, twice

(Chapter 8; page 172; line 9) Replace the second “is” by “in”

(Chapter 8; page 172; line -9) Replace 6 by 7 and Helley by Helly

(Chapter 8; page 173; line 11) replace 5 by 6

(Chapter 8; page 173) In the Remark, remove the comma in the first line. Also, add the assumption that $m(E) < \infty$, which is necessary in this version of the Dunford-Pettis Theorem. In the case $m(E) = \infty$, one needs to assume tightness and uniform integrability.

(Chapter 8; page 174) In Problem 33, replace $1 \leq p$ by $1 < p$

(Chapter 8; page 174) In Problem 36, replace Helley by Helly

(Chapter 8; page 176) In the first Example, replace “Let” by “For” and insert “a ” before “non-negative”

(Chapter 8; page 176) In the third Example, replace “a measurable set ” by “of finite measure”

(Chapter 8; page 178; line -6) replace the first “in ” by “on”

(Chapter 8; page 178; line -2) replace “ L^q ” by “ L^p ”

(Chapter 8; page 178; last line) Replace

$$\sum_{n=1}^{\infty} |f_n| \text{ with } |f_1| + \sum_{n=1}^{\infty} |f_{n+1} - f_n|$$

(Chapter 8; page 179; line 1) Replace “It is clear that” by “Since $f_n = f_1 + \sum_{k=1}^{n-1} [f_{k+1} - f_k]$, by the triangle inequality, “

(Chapter 8; page 179; line 3) Replace 32 by 33

(Chapter 8; page 179) In Problem 41, change $1 \leq p$ to $1 < p$ and, in the last line, \geq to \leq

(Chapter 8; page 180) In Problem 42, replace “a measurable set ” by “of finite measure” and remove “Let $\{f_n\}$ be a sequence in $L^{p_1}(E)$.”

(Chapter 8; page 180) In Problem 43, change $1 \leq p$ to $1 < p$

Chapter 9

(Chapter 9; page 201; line -2) Remove the comma in the statement of the Lemma

(Chapter 9; page 203 Problem 65 should be put before Problem 45

Chapter 10

(Chapter 10; page 207) Replace “The” by “the” in the statement of the lemma

(Chapter 10; page 210; line 2) Replace 7 by 8 and Helley by Helly

(Chapter 10; page 213) Extra right parenthesis in displayed equations (9) and (10)

(Chapter 10; page 213) In the line after (9), replace $E_{m,N}$ by $E(m, N)$

(Chapter 10; page 215; line 2) insert a space before uncountable

Chapter 11

(Chapter 11; page 223; line 9) Replace second X by E

(Chapter 11; page 236) Problem 45, part (ii). As Bruce Blackadar observed, it is necessary to also assume that the function f is continuous: otherwise the result is false.

Chapter 12

(Chapter 12; page 245; line -7) Remove“(not necessarily closed)” ; indeed, these sets are closed.

(Chapter 12; page 249; line 12) Remove“by possibly multiplying g by a positive number” ;

(Chapter 12; page 250; line 11) replace $n > 1$ by n

(Chapter 12; page 251) It should be Borsuk’s Theorem not Riesz’s Theorem

Chapter 13

(Chapter 13; page 265; line -10) Remove the second “of ”

(Chapter 13; page 267) Remove the box at the end of the Remark

(Chapter 13; page 269; line -11) Remove the second “for all ”

(Chapter 13; page 270) In Problem 40 interchange L^1 and L^∞

Chapter 14

(Chapter 14; page 272) On the second line of Proposition 2, insert $x \neq 0$

(Chapter 14; page 272) On the fifth line of the proof of Proposition 2, insert $x \neq 0$

(Chapter 14; page 273; line 10) Replace x by x_k

(Chapter 14; page 275; line -10) Replace X^{**} X^*

(Chapter 14; page 279) In the second Example, replace c_0 by c throughout.

(Chapter 14; page 282) In Problem 28, replace the second “functional ” by “function”

(Chapter 14; page 283; line 10) Replace Helley by Helly

(Chapter 14; page 293) In Theorem 30, remove the assumption that f is bounded below; it is unnecessary.

(Chapter 14; page 296; line 19) Remove the comma

(Chapter 14; page 296; line -2) Replace Weirstrass by Weierstrass

Chapter 15

(Chapter 15; page 298) Replace Helley by Helly, twice

(Chapter 15; page 300) Replace Helley by Helly, twice

Chapter 16

(Chapter 16; page 308) Change Helley to Helly

(Chapter 16; page 314) Change Helley to Helly

(Chapter 16; page 314; line 12) Change 14 to 13

(Chapter 16; page 314; line 14) Change 14 to 13

Chapter 17

(Chapter 17; page 340; line -7) Replace 52 with 53

(Chapter 17; page 341) In Problem 4 (i) replace \mathcal{M} by \mathcal{B} . In Problem 5 (ii) remove “and $E_2 \sim E_1 \in \mathcal{M}$ ”

(Chapter 17; page 349; line -9) Replace 6 by 7

(Chapter 17; page 351; line 3) Replace “integrals” by “intervals”

(Chapter 17; page 355; line -10) Replace \mathcal{S} by \mathcal{S}'

Chapter 18

(Chapter 18; page 359; line -14) Remove the comma after “continuity”

(Chapter 18; page 365) Replace Problem 11 by: Prove Corollary 7.

(Chapter 18; page 365) Before the second $1/2^n$ insert \leq

(Chapter 18; page 366; line 19) In the definition: First, replace E by X under the second integral. Second, replace f by ψ in the last integral

(Chapter 18; page 368; line 19)

(Chapter 18; page 368) In the second displayed equation, replace E_n by X_n

- (Chapter 18; page 372; line -2) Insert a period before “Conversely”
- (Chapter 18; page 377; line -9) Integrate over X , not over E , twice
- (Chapter 18; page 383) The font of the example needs changing
- (Chapter 18; page 384) On the last line, replace X_0 by X_+ , the second time it occurs
- (Chapter 18; page 387; line -1) Insert f after the integral sign
- (Chapter 18; page 391; line -1) Replace \mathcal{M} by \mathcal{M}/\simeq
- (Chapter 18; page 392) In the first displayed equation, replace μ by ν , twice.
- (Chapter 18; page 392) In the second displayed equation: replace ρ_μ by μ and replace μ by ν .
- (Chapter 18; page 393) In Problem 61, also assume $\mu(A \cup B) < \infty$ and the two sets are equivalent not equal.
- (Chapter 18; page 393) In Problem 66, also assume $\nu(X) < \infty$.

Chapter 19

- (Chapter 19; page 394) In the title of Section 19.2 and the corresponding page headings replace $1 \leq p \leq \infty$ by $1 \leq p < \infty$
- (Chapter 19; page 394) In the title of Section 19.4 and the corresponding page headings replace $1 < p < \infty$ by $1 < p < \infty$
- (Chapter 19; page 394) In the preamble, interchange the fifth and sixth sentences and replace “third” by “fourth”
- (Chapter 19; page 396; line 9) Replace fg by $f \cdot g$
- (Chapter 19; page 397; line 1) Replace “Minkowski” by “Holder ”
- (Chapter 19; page 399; line) On line 7 and 17, replace $|f|^p$ by $|f_n|^p$
- (Chapter 19; page 399) In the title of Section 19.2, replace $\leq p$ by $< p$ in the section title
- (Chapter 19; page 403; line 11) Replace $L^p(X, \mu)$ by $L^p(X, \mu)$
- (Chapter 19; page 403; line -5) Replace $L^p(X, d\mu)$ by $L^p(X, \mu)$
- (Chapter 19; page 407) In the title of Section 19.4 and the corresponding page headings replace $1 < p < \infty$ by $1 < p < \infty$
- (Chapter 19; page 407; line -4) Replace $\int_a^b g \cdot h$ by $\int_X g \cdot h d\mu$
- (Chapter 19; page 409) In problem 16, replace l by ℓ

Chapter 20

(Chapter 20; page 414; line -9) Replace ‘measurable rectangle’ by ‘measurable rectangle provided $\mu(A) < \infty$ and $\nu(B) < \infty$.’

(Chapter 20; page 417; line -8) Replace (5) by (2)

(Chapter 20; page 417; line -4) Replace “proposition” by “theorem”

(Chapter 20; page 418; line -8) Replace (5) by (2)

(Chapter 20; page 420; line -15) Put $X \times Y$ under the integral sign

(Chapter 20; page 420; line -2) Replace “in” by “is”

(Chapter 20; page 420; line -8) Insert “on” before $X \times Y$

(Chapter 20; page 422) In the first line of Problem 4, replace “general” by “complete”

(Chapter 20; page 423) In Problem 6: (i) Remove “(is an $\mathcal{R}_{\sigma\delta}$, in fact)”; (ii) Replace “D” by “ Δ , then ”

(Chapter 20; page 423) In Problem 7, replace “general” by “complete”

(Chapter 20; page 423) In Problem 10, replace “, then” by “and”

(Chapter 20; page 425; line -6) Replace “on” by “of”

(Chapter 20; page 425; line -3) Replace “vol” by “ μ integral”

(Chapter 20; page 427; line -3) At the end of the first proof replace $I_{k,n}$ by $I_{k,m}$, four times.

(Chapter 20; page 427) In the statement of Theorem 13, replace “of” by “be”

(Chapter 20; page 430; line 10) Replace $\mathcal{L}(R^n)$ by R^n

(Chapter 20; page 432; line 3) Replace $1/|\alpha|$ by $|\alpha|$

(Chapter 20; page 432; line 13) Replace $1/|c|$ by $|c|$

(Chapter 20; page 438) On the line before (30), replace \int_a^b by $\int_{a,b]$

(Chapter 20; page 440) Problem 40: remove the first “bounded”

(Chapter 20; page 441) Insert a period at the end of the second displayed equation.

Chapter 21

(Chapter 21; page 447) In the footnote, replace ‘our view ’ by ‘view of ’

(Chapter 21; page 461) Remove the first comma in the statement of Problem 40

Chapter 22

(Chapter 22; page)

Chapter 23

(Chapter 23; page)